Security & Economics — Part 10

Review

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Outline

Outline

Idea of the course

1 - Benefits from security investment

2 - External view of security investment

3 - Auctions and sponsored search

4 - Network externalities and information asymmetry

5 - Social welfare and social choice

Idea of the course

Security = Economy

- A security procedure is effective
  only if it is cost effective.
- An asset is an asset
  only if it can be secured.

Therefore we studied

economics of security:

- incentives for the attackers
- costs for the defenders

security of market:

- not just buying and selling
- but also cheating and stealing

Tasks

economics of security:

- Protect the organizations from the world

security of market:

- Protect the world from the organizations

Methods

economics of security:

- pricing and costing security investments

security of market:

- security of pricing and costing

Employment view

economics of security:

- security manager / CIO
- accounting tools for market of security

security of market:

- mechanism designer
- security tools for network economy

Outline

The employment view of the course

- security manager: lectures 2–3
  - accounting tools for the market of security
- mechanism designer: lectures 4–8
  - security tools for network economy

The employment view of the course

- **security manager: lectures 2–3**
  - **accounting tools for the market of security**
- mechanism designer: lectures 4–8
  - security tools for network economy

Accounting for security investments

Question

Given the costs and the benefits, how do we calculate the value of security investments?

Example 1

- On January 1, 2012, ToySec buys a firewall for £200,000.
- During the year 2012, ToySec accumulates
  - firewall operating costs of £100,000, and
  - security benefits of £400,000

Basic accounting: Value

Net cash flow (NCF)

2012-01-01 - £200K

2013-01-01 £400K - £100K = £300K

Value (V) = total cash flow

2012-01-01 - £200K

2013-01-01 - £200K + £300K = £100K

Example 1'

- On January 1, *2013*, ToySec buys a firewall for £200,000.
- During the year *2013*, ToySec *is expected* to accumulate
  - firewall operating costs of £100,000, and
  - security benefits of £400,000

Basic accounting: *Future* Value

Net cash flow (NCF)

2013-01-01 - £200K

2014-01-01 £400K - £100K = £300K

Future value (FV) = total *expected* cash flow

2013-01-01 - £200K

2014-01-01 - £200K + £300K = £100K

Example 1 again

time	1-1-2012	1-1-2013
security benefit	0	£400,000
security cost	£200,000	£100,000

Example 1 again

time	1-1-2012	1-1-2013
security benefit	0	£400,000
security cost	£200,000	£100,000

$$\text{annual return on investment} = \frac{\text{investment profit}}{\text{investment cost}}$$

Example 1 again

time	1-1-2012	1-1-2013
security benefit	0	£400,000
security cost	£200,000	£100,000

$$\begin{aligned}\text{annual return on investment} &= \frac{\text{investment profit}}{\text{investment cost}} \\ &= \frac{(400,000 - 100,000)}{200,000} \\ &= 150\%\end{aligned}$$

Concept 1: Annual return on investment (AROI)

Definition

Annual eturn on investment (AROI) is the accounting concept obtained by dividing

- investment profit in a given year, obtained by subtracting
  - the costs C_1 from
  - the benefits B_1

  with
- investment costs C_0, needed to generate the profit

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  with
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$$\text{AROI} = \frac{B_1 - C_1}{C_0}$$

Concept 1: Annual return on investment (AROI)

Decision rule

AROI $> 100\%$ — accept the investment

AROI $< 100\%$ — reject the investment

AROI $= 100\%$ — offers no grounds for a decision

Example 1 yet again

time	1-1-2012	1-1-2013
security benefit	0	£400,000
security cost	£200,000	£100,000

$$\text{AROI} \quad = \quad \frac{(400,000 - 100,000)}{200,000} \quad = \quad 150\%$$

$\Longrightarrow$ invest!

Example 2

time	1-1-2012	1-1-2013
security benefit	0	£300,000
security cost	£250,000	£100,000

$$\text{AROI} = \frac{(300,000 - 100,000)}{25,000} = 80\%$$

$\Longrightarrow$ do not invest!

Example 3

time	1-1-2012	1-1-2013
security benefit	0	£300,000
security cost	£200,000	£100,000

$$\text{AROI} = \frac{(300,000 - 100,000)}{200,000} = 100\%$$

$\Longrightarrow$ use a different accounting concept?

Accounting of security investments

Question

How do we calculate return on multi-period investments?

Example 4

- On January 1, 2013, ToySec buys an intrusion detection system for £200,000.
- During the year 2013 ToySec is expected to accumulate
  - firewall operating costs of £100,000, and
  - security benefits of £400,000

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- During the year 2014 ToySec is expected to accumulate
  - firewall operating costs of £100,000, and
  - security benefits of £450,000

Example 4

time	1-1-2013	1-1-2014	1-1-2015
security benefit	0	£400,000	£450,000
security cost	£200,000	£100,000	£100,000

Example 4

time	1-1-2013	1-1-2014	1-1-2015
security benefit	0	£400,000	£450,000
security cost	£200,000	£100,000	£100,000

$$\text{simple return on investment} = \frac{\text{total investment profit}}{\text{total investment cost}}$$

$$= \frac{(0 - 200) + (400 - 100) + (450 - 100)}{200 + 100 + 100}$$

$$= \frac{450}{400} = 112.5\%$$

Concept 1': Simple return on investment (SROI)

Definition

Simple eturn on investment (SROI) is the accounting concept obtained by dividing

- total investment profit in a given period, obtained by subtracting
  - total costs $\sum_i C_i$ from
  - total benefits $\sum_i B_1$

  with
- total costs $\sum_i C_i$, needed to generate the profit

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  - total benefits $\sum_i B_1$

  with
- total costs $\sum_i C_i$, needed to generate the profit

$$\text{SROI} = \frac{\sum_i B_i - \sum_i C_i}{\sum_i C_i}$$

Concept 1': Simple return on investment (SROI)

Decision rule

The more the better

Accounting of security investments

Question

What is the net present value of multi-period investments?

Example 4 again

- On January 1, 2013, ToySec buys an intrusion detection system for £200,000.
- During the year 2013 ToySec is expected to accumulate
  - firewall operating costs of £100,000, and
  - security benefits of £400,000

Example 4 again

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Example 4 again

- On January 1, 2013, ToySec buys an intrusion detection system for £200,000.
- During the year 2013 ToySec is expected to accumulate
  - firewall operating costs of £100,000, and
  - security benefits of £400,000
- During the year 2014 ToySec is expected to accumulate
  - firewall operating costs of £100,000, and
  - security benefits of £450,000
- ToySec's *cost of capital* is 15%.

Concept 2: Net Present Value (NPV)

Definition

The net present value (NPV) of an investment is the sum of

- the annual values of the investment, obtained by subtracting for each year t
  - the costs C_t from
  - the benefits B_t
- discounted by the annual *cost of capital* k
  - which is the minimal rate of return that every project needs to earn in order for the organization to break even.

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- discounted by the annual *cost of capital* k
  - which is the minimal rate of return that every project needs to earn in order for the organization to break even.

$$\text{NPV} = \sum_{t=0}^{n} \frac{B_t - C_t}{(1+k)^t}$$

where usually $B_0 = 0$, except when there are instant benefits.

Concept 2: Net Present Value (NPV)

Decision rule

$\text{NPV} > 0$ — accept the investment

$\text{NPV} < 0$ — reject the investment

$\text{NPV} = 0$ — offers no grounds for a decision

Example 4

time	1-1-2013	1-1-2014	1-1-2015
security benefit	0	£400,000	£450,000
security cost	£200,000	£100,000	£100,000
cost of capital	15%		

Example 4

time	1-1-2013	1-1-2014	1-1-2015
security benefit	0	£400,000	£450,000
security cost	£200,000	£100,000	£100,000
cost of capital	15%		

$$\begin{aligned} NPV &= -200,000 + \frac{300,000}{1.15} + \frac{350,000}{1.15^2} \\ &= -200,000 + 260,870 + 264,650 \\ &= 325,520 \end{aligned}$$

$\Longrightarrow$ invest!

Example 5

time	1-1-2013	1-1-2014
security benefit	0	£400,000
security cost	£280,000	£100,000
cost of capital	15%	

$$
\begin{aligned}
\text{NPV} &= -280,000 + \frac{300,000}{1.15} \\
&= -280,000 + 260,870 \\
&= -19,130
\end{aligned}
$$

$\Longrightarrow$ do not invest!

Example 6

time	1-1-2013	1-1-2014
security benefit	0	£400,000
security cost	£200,000	£100,000
cost of capital	50%	

$$\begin{aligned} \text{NPV} &= -200,000 + \frac{300,000}{1.5} \\ &= -200,000 + 200,000 \\ &= 0 \end{aligned}$$

$\Longrightarrow$ take risk aversion into account?

Accounting of security investments

Question

Is it better to invest in security or in something else?

Concept 3: Internal rate of return (IRR)

Definition

The internal rate of return (IRR) of an investment is the discount rate which makes the net present value of a security investment equal to 0.

Concept 3: Internal rate of return (IRR)

Definition

The internal rate of return (IRR) of an investment is the discount rate which makes the net present value of a security investment equal to 0.

$$0 \;=\; \sum_{t=0}^{n} \frac{B_t - C_t}{(1 + \mathrm{IRR})^t}$$

where usually $B_0 = 0$, except when there are instant benefits.

Concept 3: Internal rate of return (IRR)

Decision rule

Suppose that an investment A has a rate of return k_A.

$\mathrm{IRR} > k_A$ — invest in security (not in A)

$\mathrm{IRR} < k_A$ — do not invest in security (invest in A)

$\mathrm{IRR} = k_A$ — consider other preferences

Example 7

- On January 1, 2013, ToySec buys an intrusion detection system for 280,000 .
- During the years 2014 and 2015 ToySec is expected to accumulate
  - firewall operating costs of £100,000 , and
  - security benefits of £400,000
- ToySec's cost of capital is 15%.

Example 7

time	1-1-2013	1-1-2014	1-1-2015
security benefit	0	£400,000	£400,000
security cost	£280,000	£100,000	£100,000
rate of return of A	15%		

Example 7

time	1-1-2013	1-1-2014	1-1-2015
security benefit	0	£400,000	£400,000
security cost	£280,000	£100,000	£100,000
rate of return of *A*	15%		

$$0 = -280,000 + \frac{300,000}{1+\text{IRR}} + \frac{300,000}{(1+\text{IRR})^2}$$

$$\Longrightarrow \quad \text{IRR} = 70.12\% > 15\% = k_A$$

$\Longrightarrow$ invest in security!

Example 8

time	1-1-2013	1-1-2014
security benefit	0	£400,000
security cost	£280,000	£100,000
cost of capital	15%	

$$0 = -280,000 + \frac{300,000}{1 + \mathrm{IRR}}$$

$$\Longrightarrow \mathrm{IRR} = 7.14\% < 15\% = k_A$$

$$\Longrightarrow$$ invest in *A*!

Outline

External view of security investments

Question

How does my neighbor's security influence my own security investment?

Games model interdependencies

Definition

An *n-player game* is an *n*-tuple of utility functions $u = \langle u_i \rangle_{i=1}^n : \prod_{i=1}^n A_i \to \mathbb{R}^n$ where

- $i = 1, 2, \ldots, n$ are the *players*
- A_i is the set of moves available to the player i
- $u_i : A \to \mathbb{R}$ is i's utility

Utility

Terminology

A function $u : A \to \mathbb{R}$ is called *utility* when it is used to express a preference relation.

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Remark

The relation $\succ \subseteq A \times A$ defined

$$a \succ b \iff u(a) > u(b)$$

is the induced *preference relation*.

Bimatrix presentation of 2-player games

- $n = 2$
- $A_1 = \{U, D\}$
- $A_2 = \{L, R\}$
- $u = \langle u_1, u_2 \rangle : A_1 \times A_2 \to \mathbb{R}^2$

	L	R
U	$u_1(U,L)$, $u_2(U,L)$	$u_1(U,R)$, $u_2(U,R)$
D	$u_1(D,L)$, $u_2(D,L)$	$u_1(D,R)$, $u_2(D,R)$

Game 1: Studying together

- $n = 2$
- $A_1 = A_2 = M = \{\text{work}, \text{goof}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	work	goof
work	2 / 2	3 / −2
goof	−2 / 3	−1 / −1

Game 1: Studying together

- $n = 2$
- $A_1 = A_2 = M = \{\text{work}, \text{goof}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	work	goof
work	b, b	d, a
goof	a, d	c, c

$$a \succ b \succ c \succ d$$

Game 2: Prisoners' Dilemma

- $n = 2$
- $A_1 = A_2 = M = \{\text{deny}, \text{confess}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	deny	confess
deny	-1, -1	-11, 0
confess	0, -11	-10, -10

Notation and terminology

- players: $i = 1, 2, \ldots, n$
- moves: $s_i, t_i \in A_i$
- profiles $s = \langle s_1, \ldots s_n \rangle \in A = \prod_{i=1}^{n} A_i$
- $s_{-k} \in A_{-k} = \prod_{\substack{i=1 \\ i \neq k}}^{n} A_i$

Best response strategy

Definition

A *best response strategy* for a player k in a given game $u : A \to \mathbb{R}^n$ is a relation

$$BR_i \quad \subseteq \quad A_{-k} \times A_k$$

such that

$$a_{-k} \; BR_k \; a_k \quad \Longleftrightarrow \quad \forall x_k \in A_k.\; u_k(x_k, a_{-k}) \leq u_k(a_k, a_{-k})$$

Best response profile

Definition

A *best response profile* for a given game $u : A \to \mathbb{R}^n$, where $A = \prod_{i=1}^n A_i$ is a relation

$$BR \quad \subseteq \quad A \times A$$

such that

$$s \; BR \; t \quad \iff \quad \forall k.\; s_{-k} \; BR_k \; t_k$$

Nash equilibrium

Definition

A *(Nash) equilibrium* for a given game $u : A \to \mathbb{R}^n$, where $A = \prod_{i=1}^{n} A_i$ is a profile $s \in A$ such that

$$s \; BR \; s$$

Nash's Theorem

Theorem (Nash)

Every game between finitely many players, with finitely many pure moves has a Nash equilibrium, provided that mixed strategies are allowed.

From payoffs to Nash equilibrium

$$\dfrac{\dfrac{\prod_{i=1}^n A_i \xrightarrow{\varrho} \mathbb{R}^n}{A_{-i} \xrightarrow{BR_i} A_i}}{\dfrac{\prod_{i=1}^n A_i \xrightarrow{\langle BR_i \circ \pi_i \rangle} \prod_{i=1}^n A_i}{1 \xrightarrow{BR_i^\bullet} A_i}\ \textbf{Fix}}$$

Security Investment Game

- $n = 2$
- $A_1 = A_2 = M = \{\text{invest}, \text{don't}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$
- C = cost of the investment
- L = value under threat
- v = vulnerability: probability of successful attack
- w = total transferred vulnerability
  - received from the neighbors

Security Investment Game

	invest	don't
invest	$-C$ $-C$	$-vL$ $-C - wL$
don't	$-C - wL$ $-vL$	$-vL - (1 - v)wL$ $-vL - (1 - v)wL$

Security Investment Game

- if $C < v(1-w)L$ then
  - ⟨invest, invest⟩ is dominant equilibrium
- if $v(1-w)L < C < vL$ then
  - there is no dominant equilibrium
  - ⟨invest, invest⟩ and ⟨don't, don't⟩ are Nash equilibria
- if $vL < C$ then
  - ⟨don't, don't⟩ is dominant equilibrium

Outline

The employment view of the course

- security manager: lectures 2–3
    - accounting tools for the market of security
- mechanism designer: lectures 4–8
    - security tools for network economy

The employment view of the course

- security manager: lectures 2–3
  - accounting tools for the market of security
- **mechanism designer: lectures 4–8**
  - **security tools for network economy**

Two forms of social choice

- market: aggregate utilities (quantitative)
- voting: aggregate preferences (qualitative)

Two forms of social choice

- market: Lectures 4–7
- voting: Lecture 8

Introduction

Market is a system of exchange protocols

- compute the prices
- regulate the exchange

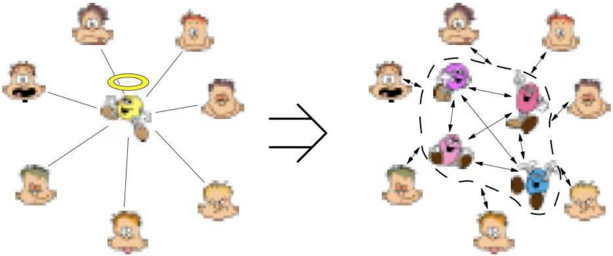
Introduction

Market is a system of exchange protocols

- compute the prices
- regulate the exchange

We focus on computing the prices.

Market as computation



Market is a multi-party computation of the prices

Auction as market organized by

- a seller: supply auction
- a buyer: procurement auction

Market computation modeling

- Market security

Market computation modeling

- Market security

$\Uparrow$

- Auction security

Market computation modeling

- Market security

  $\Uparrow$

- Auction security

  $\Uparrow$

- Games and mechanisms

Auction protocols: Requirement

Given a set of sellers and a set of buyers with *private utilities*, auction protocols are designed to

- maximize seller's revenue: supply auctions
- minimize buyer's cost: procurement auctions

Auction protocols: Problem

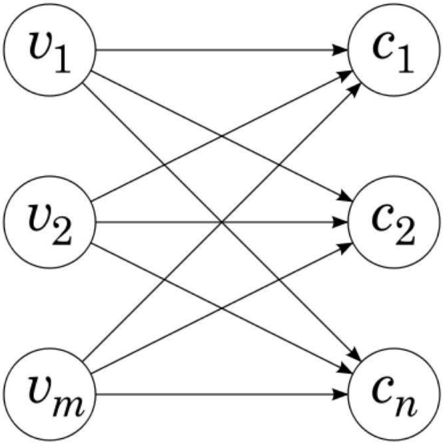
- To maximize revenue, the sellers must keep their utility private
- To minimize cost, the buyers must keep their utility private

Auction protocols: Goal

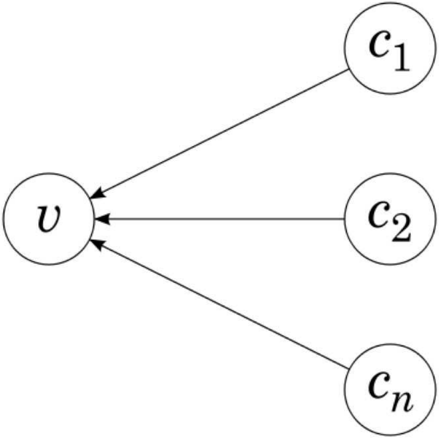
Definition

An auction mechanism is said to be *incentive compatible* if it elicits truthful bidding, i.e. provides the bidders with an incentive to bid their true valuations.

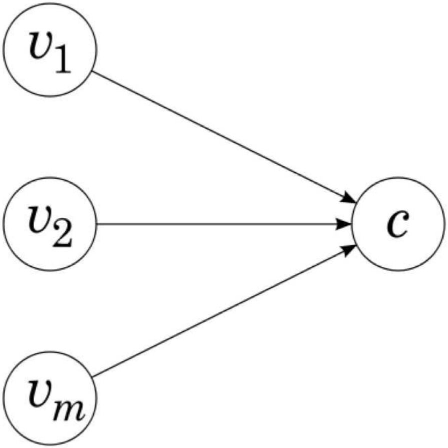
Multi-item auction



Single-user procurement (demand) auction



Single-item (supply) auction



Taxonomy of single item auctions

	interactive	sealed bid
strategic	descending	first price
truthful	ascending	second price

Equivalence of interactive and sealed bidding

- with the ascending auction, the highest bidder pays second highest bidder's valuation
- with the descending auction, the highest bidder pays the first announcememt below his own valuation

Modeling auctions as games

- players: $i = 1, 2, \ldots, n$
- moves: $A_i = \mathbb{R}$
- payoffs: $u = \langle u_i \rangle_{i=1}^{n} : \mathbb{R}^n \to \mathbb{R}^n$

$$u_i(b) \quad = \quad \tau_i(b) \cdot \big(v_i - p(b)\big)$$

where

- $b = \langle b_i \rangle_{i=1}^{n} \in \mathbb{R}^n$ is the bidding profile

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  - $b = \langle b_i \rangle_{i=1}^n \in \mathbb{R}^n$ is the bidding profile
  - $v = \langle v_i \rangle_{i=1}^n \in \mathbb{R}^n$ is the valuation profile

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- $p(b)$ is the winning price for the bids b

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- $b = \langle b_i \rangle_{i=1}^{n} \in \mathbb{R}^n$ is the bidding profile
- $v = \langle v_i \rangle_{i=1}^{n} \in \mathbb{R}^n$ is the valuation profile
- $p(b)$ is the winning price for the bids b
- $\tau_i(b) = \begin{cases} 1 & \text{if } i = \omega(b) \\ 0 & \text{otherwise} \end{cases}$ and

  $\omega(b) = \min\{j \leq n \mid \forall k.\ b_k \leq b_j\}$ is the auction winner

Modeling auctions as games

Assumption

Without loss of generality, we assume that the bid vector $b = \langle b_1, b_2, \ldots, b_n \rangle$ is arranged in descending order

$$b_1 \geq b_2 \geq b_3 \geq \cdots \geq b_n$$

Modeling auctions as games

Assumption

Without loss of generality, we assume that the bid vector $b = \langle b_1, b_2, \ldots, b_n \rangle$ is arranged in descending order

$$b_1 \geq b_2 \geq b_3 \geq \cdots \geq b_n$$

Since only one bidder wins, and the priority of equal bidders is resolved lexicographically, nothing is lost if the equal bidders are ignored, so we assume that the bid vector is strictly descending

$$b_1 > b_2 > b_3 > \cdots > b_n$$

Modeling auctions as games

Definition

The winning price is

- in the first price auction:

$$p_1(b) \quad = \quad b_1$$

- in the second price auction:

$$p_2(b) \quad = \quad b_2$$

Rational bidding in second price auctions

Proposition

The truthful bidding

$$\overline{b}_i = v_i$$

is the dominant strategy for the second price sealed bid auctions.

Rational bidding in first price auctions

Proposition

In a first price sealed bid auction

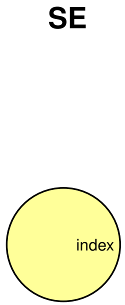
- with n players,
- with the valuations v_i uniformly distributed in an interval $[0, x]$

the Nash equilibrium consists of the bids

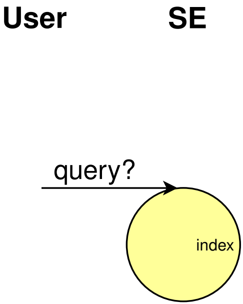
$$\overline{b}_i = \beta(v_i) = \frac{n-1}{n} \cdot v_i$$

where $\beta : \mathbb{R} \to \mathbb{R}$ denotes the equilibrium strategy used by all players.

Sponsored search business model



Sponsored search business model



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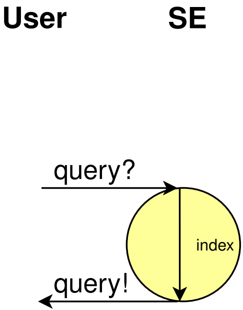
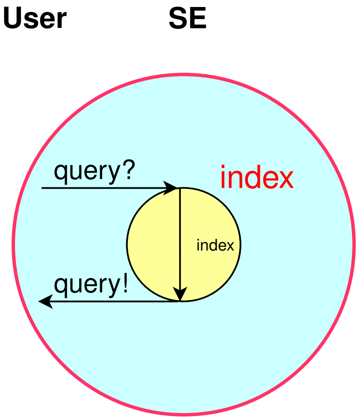


Sponsored search business model



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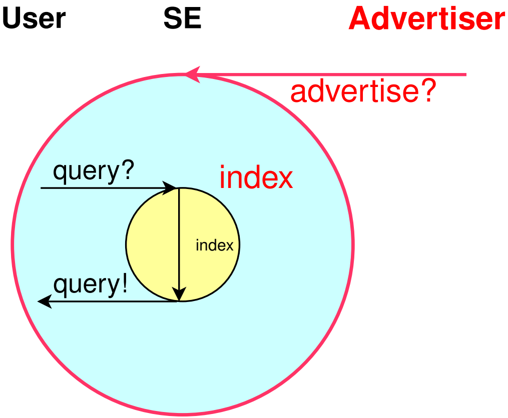

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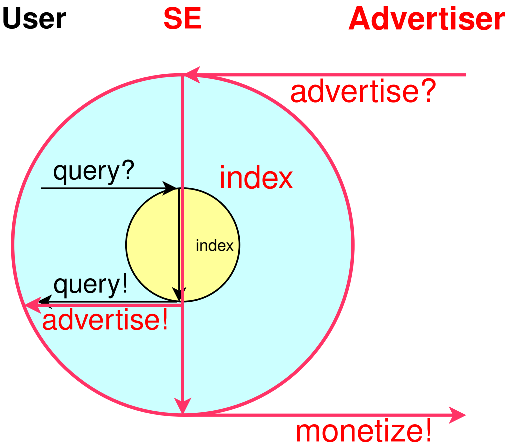
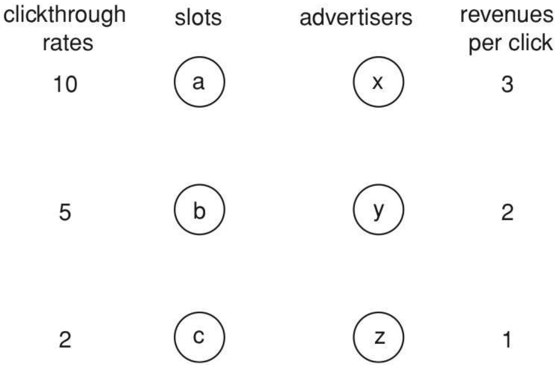


Sponsored search business model

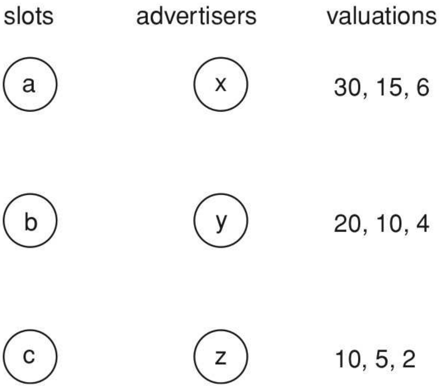
Position auction

Advertisers **bid for positions** among the search results.

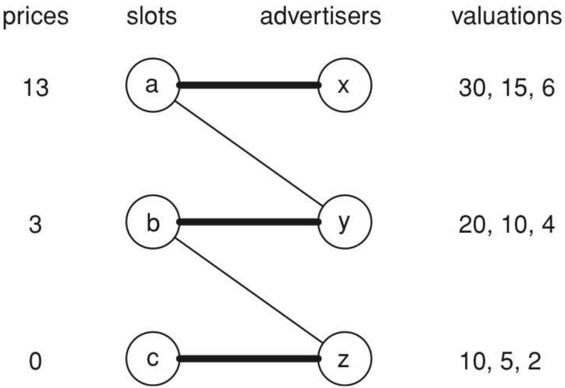
Sponsored search auction setting



Sponsored search as a matching problem



Sponsored search as a market



Market mechanism

- n buyers, n item
  - take $n = \{0, 1, \ldots, n-1\}$

Market mechanism

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- buyers valuations per item $v = (v_{ij})_{n \times n}$

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- matching $\sigma_{vp} : n \to n$ assigns item $\sigma_{vp}(i)$ to i

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  - take $n = \{0, 1, \ldots, n-1\}$
- buyers valuations per item $v = (v_{ij})_{n \times n}$
- item prices $p = (p_i)_n$
- matching $\sigma_{vp} : n \to n$ assigns item $\sigma_{vp}(i)$ to i
- i's utility $u_i \in \mathbb{R}$ is

$$u_i = v_{i\sigma_{vp}(i)} - p_{\sigma_{vp}(i)}$$

Goal of the market mechanism

Maximize social welfare, i.e. buyers' total payoff

$$
\begin{aligned}
U(v,p) &= \sum_{i \in n} u_i \\
&= \sum_{i \in n} v_{i\sigma_{vp}(i)} - p_{\sigma_{vp}(i)} \\
&= \sum_{i \in n} v_{i\sigma(i,v)} - P
\end{aligned}
$$

where $P = \sum_{i<n} p_i$

Markets respect preference

To maximize utility, $\sigma_{vp} : n \to n$ maximizes valuations

$$v_{i\sigma(i,v)} \geq v_{ij}$$

Position auction mechanism

- n bidders, n positions

Position auction mechanism

- n bidders, n positions
- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
  - bidders' valuations per click $w = (w_i)_n$
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  - bidders' valuations per click $w = (w_i)_n$
  - position click-through rates $r = (r_j)_n$
- bidders bid $b = (b_i)_n$
- price per position $\pi_{ij}(b) = p_i(b) \cdot r_j$ where
  - price per click $p(b) = \big(p_i(b)\big)_n$

Position auction mechanism

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- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
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  - price per click $p(b) = \big(p_i(b)\big)_n$
- matching $\tau : n \times \mathbb{R}^n \to n$ assigns item $\tau(i, b)$ to i

Position auction mechanism

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  - price per click $p(b) = \big(p_i(b)\big)_n$
- matching $\tau : n \times \mathbb{R}^n \to n$ assigns item $\tau(i, b)$ to i
- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) \;=\; v_{i\tau(i,b)} - \pi_{i\tau(i,b)}(b) \;=\; \big(w_i - p_i(b)\big) \cdot r_{\tau(i,b)}$$

Goal of the position auction mechanism

Maximize seller's revenue

$$
\begin{aligned}
P(b) &= \sum_{i<n} \pi_{i_{\tau(i,b)}}(b) \\
&= \sum_{i<n} p_i(b) \cdot r_{\tau(i,b)}
\end{aligned}
$$

where

- all p_i grow with b
- bidder i bids b_i to maximize $u_i(b)$.

Position auctions respect preference

To maximize $p_i(b)$ with $u_i(b)$ always use

- $\tau(i,b) < \tau(j,b) \implies b_i \geq b_j$, i.e.
- $\tau(i,b) = j$ if b_i is j-th largest entry in b

Assumption

- The bidders are ordered by their bids

$$b_1 > b_2 > b_3 > \cdots > b_n$$

- The positions are ordered by click-through rates

$$r_1 > r_2 > r_3 > \cdots > r_n$$

Generalized Second Price Auction

- n bidders, n positions
- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
  - bidders' valuations per click $w = (w_i)_n$
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Generalized Second Price Auction

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- bidders bid $b = (b_i)_n$
- price per click $p_i(b) = b_{i+1}$

Generalized Second Price Auction

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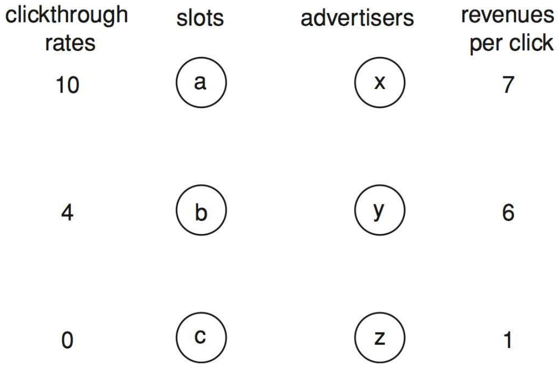
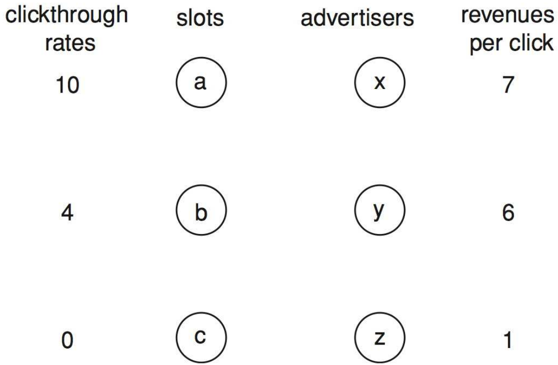
- price per click $p_i(b) = b_{i+1}$
- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) \ = \ \left(w_i - b_{i+1}\right) \cdot r_i$$

Is GSP incentive compatible?



Is GSP incentive compatible?



- with truthful bid: $u_x(7,6,1) \; = \; (7-6)\cdot 10 \; = \; 10$
- with untruthful bid: $u_x(5,6,1) \; = \; (7-1)\cdot 4 \; = \; 24$

Idea: Vickrey, Clarke, Groves

- Each bidder should pay the cost that their bid incurs on social welfare
  - i.e., the sum of the losses that they cause to other bidders

Vickrey-Clarke-Groves Auction

Notation

- B — set of bidders
- S — set of sellers (items)
- $v = (v_{ij})_{B \times S}$ — bidders' valuations
- V_B^S — maximal total valuation

Vickrey-Clarke-Groves Auction

Notation

- B — set of bidders
- S — set of sellers (items)
- $v = (v_{ij})_{B \times S}$ — bidders' valuations
- V_B^S — maximal total valuation

Remark

- If $\#B < \#S$, then add $\#S - \#B$ bidders with all valuations 0
- If $\#B > \#S$, then add $\#B - \#S$ sellers valued 0 by all.

Vickrey-Clarke-Groves Auction

Remember the assumption

- The bidders are ordered by their bids

$$b_1 > b_2 > b_3 > \cdots > b_n$$

- The positions are ordered by click-through rates

$$r_1 \geq r_2 \geq r_3 \geq \cdots \geq r_n$$

Vickrey-Clarke-Groves Auction

- n bidders, n positions
- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
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- bidders bid $b = (b_i)_n$
- price per item $\pi_{ij}(b) = V^S_{B\setminus i} - V^{S\setminus j}_{B\setminus i}$

Vickrey-Clarke-Groves Auction

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- price per item $\pi_{ij}(b) = V^S_{B\setminus i} - V^{S\setminus j}_{B\setminus i}$
- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) \;=\; v_{ii} - \pi_{ii}(b)$$

Vickrey-Clarke-Groves Auction

Theorem

The VCG auction is incentive compatible: truthful bidding is the unique Nash equilibrium for all players.

Vickrey-Clarke-Groves Auction

Corollary

The VCG auction maximizes social wellfare, i.e. the total utility of bidders.

Outline

Idea of the course

1 - Benefits from security investment

2 - External view of security investment

3 - Auctions and sponsored search

4 - Network externalities and information asymmetry

Self-fulfilling expectations

Market of lemons

Intrinsic values and externalities

Intrinsic values of goods are expressed through their market prices and their production costs.

Externalities are the values of goods taken by those who are neither producers nor consumers of these goods.

Examples of externalities

Positive:

- public health, security, education
- freeware, creative commons
- social adoption of shared applications

Negative:

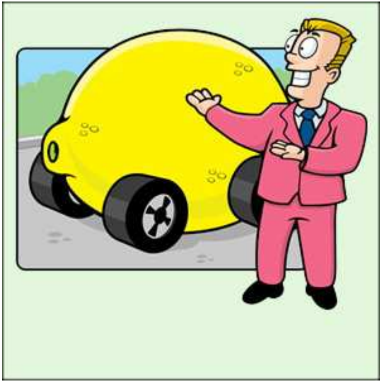
- pollution, environmental change
- exploitation of resources (e.g. fishing)
- systemic risk (e.g. in banking)
- congestion
- price increase due to demand

Self-fulfilling expectations equilibrium

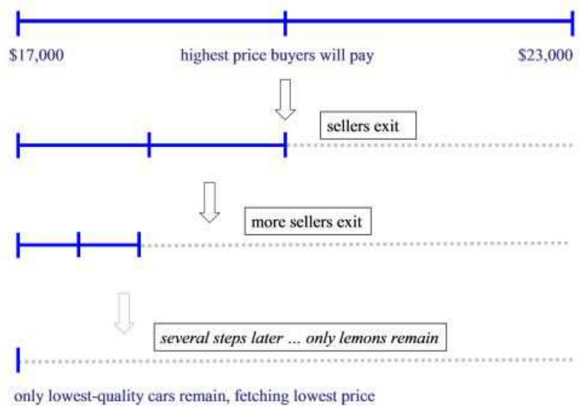
If the value of a good depends on its market adoption, then

- users' *belief* (expectation) that the good is adopted by a p-part of the market
- causes the good to be *really* adopted by a p-part of the market.

Market of lemons



Market of lemons



Akerloff's analysis

- valuations:

	good cars	lemons
sellers	x	0
buyers	$\frac{3}{2}x$	0

- **quality distribution**: q-fraction of cars is worth $\frac{qx}{2}$ on the average
- demand:

$$\#\text{buyers} \quad > \quad \#\text{cars for sale}$$

Akerloff's analysis

1. Symmetric information

- Both sellers and buyers can tell which cars are good.
- Each good car is sold for its true value.
- The lemons are unsold or given for free.
- Since #buyers $>$ #cars for sale, the market clears.

Akerloff's analysis

2. Asymmetric information: Naive buyers

- Only sellers know which cars are good.
- The buyers
  - expect the cars with $w_0 \in \left[0, \frac{3x}{2}\right]$ uniformly distributed
  - offer the average price $p_0 = \frac{3x}{4}$.
- The sellers
  - withdraw the cars with sellers' values $v \in \left(\frac{3x}{4}, x\right]$ and
  - clear the $\frac{3}{4}$ of the cars with sellers' values $v \in \left[0, \frac{3x}{4}\right]$
- The buyers
  - get the average value $w_1 = \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{3x}{2} = \frac{9x}{16}$
  - pay the average price $p_0 = \frac{3x}{4}$

Akerloff's analysis

3. Asymmetric information: Rational buyers

- Only sellers know which cars are good.
- The buyers
  - expect the cars with $w_0 \in \left[0, \frac{3x}{2}\right]$ uniformly distributed
  - offer the average price $p_0 = \frac{3x}{4}$.
- The sellers
  - withdraw the cars with sellers' values $v \in \left(\frac{3x}{4}, x\right]$ and
  - clear the $\frac{3}{4}$ of the cars with sellers' values $v \in \left[0, \frac{3x}{4}\right]$
- The buyers
  - know that the values are now $w_1 \in \left[0, \frac{3}{4} \cdot \frac{3x}{2}\right] = \left[0, \frac{9x}{8}\right]$
  - offer the average price $p_1 = \frac{9x}{16}$

Akerloff's analysis

3. Asymmetric information: Rational buyers

- Only sellers know which cars are good.
- The buyers
  - expect the cars with $w_1 \in \left[0, \frac{9x}{8}\right]$ uniform
  - offer the average price $p_1 = \frac{9x}{16}$.
- The sellers
  - withdraw the cars with sellers' values $v \in \left(\frac{9x}{16}, x\right]$ and
  - clear the $\frac{9}{16}$ of the cars with sellers' values $v \in \left[0, \frac{9x}{16}\right]$
- The buyers
  - know that the values are $w_2 \in \left[0, \frac{9}{16} \cdot \frac{3x}{2}\right] = \left[0, \frac{27x}{32}\right]$
  - offer the average price $p_2 = \frac{27x}{64}$

Akerloff's analysis

3. Asymmetric information: Rational buyers

- Only sellers know which cars are good.
- The buyers
  - expect the cars with $w_2 \in \left[0, \frac{27x}{32}\right]$ uniformly distributed
  - offer the average price $p_1 = \frac{27x}{64}$.
- The sellers
  - withdraw the cars with sellers' values $v \in \left(\frac{27x}{64}, x\right]$ and
  - clear the $\frac{27}{64}$ of the cars with values $v \in \left[0, \frac{27x}{64}\right]$
- The buyers
  - know that the values are $w_3 \in \left[0, \frac{81x}{128}\right]$
  - offer the average price $p_3 = \frac{81x}{256}$

Akerloff's analysis

3. Asymmetric information: Rational buyers

- Only sellers know which cars are good.
- $w, p \searrow 0$
- **The market collapses!**

Outline

Social choice

Kenneth Arrow's Thesis (1948, 1951)

'In a capitalist democracy there are essentially two methods by which social choices can be made:

- voting, typically used to make "political" decisions, and
- the market mechanism, typically used to make "economic" decisions.'

Social choice

Kenneth Arrow's Thesis (1948, 1951)

'. . . In the emerging democracies with mixed economic systems, Great Britain, France, and Scandinavia, the same two modes of making social choices prevail, though more scope is given to the method of voting and decisions based directly or indirectly on it and less to the rule of the price mechanism. Elsewhere in the world, and even in smaller social units within the democracies, social decisions are sometimes made by single individuals or small groups.'

Preference space

Definition

The *preference space* over a set $\mathcal{S}$ is the set $\mathbb{P}$ of all preference relations $\succ$ over $\mathcal{S}$

$$\mathbb{P} = \Big\{ \succ \subseteq \mathcal{S} \times \mathcal{S} \mid X \succ Y \succ Z \implies X \succ Z \\ \wedge (X \succ Y \vee Y \succ X) \Big\}$$

Social welfare function

Definition

For a society consisting of the players $i = 1, 2, \ldots n$, a *social welfare function (swf)* is a mapping

$$\begin{array}{rccc} \wr - \int_w & : \mathbb{P}^n & \longrightarrow & \mathbb{P} \\ & \succ & \longmapsto & \wr \succ \int_w \end{array}$$

where $\succ = \langle \overset{1}{\succ}, \overset{2}{\succ}, \ldots, \overset{n}{\succ} \rangle$

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where $\succ = \langle \overset{1}{\succ}, \overset{2}{\succ}, \ldots, \overset{n}{\succ} \rangle$

The relation $\wr \succ \int_w$ is the *aggregate preference*, or *social welfare|* induced by the profile $\succ \in \mathbb{P}^n$.

Social choice function and relation

Definition

A *social choice function (scf)* is a mapping $\wr : \int_f \mathbb{P}^n \to A$.

A *social choice relation (scr)* is a mapping $\wr : \int_r \mathbb{P}^n \to \wp A$.

Social choice function and relation

Example 1

A swf $\wr - \int_w$ always induces a scr

$$c \in \wr \succ \int_r \iff \forall x.\ c \wr \succ \int_w x$$

It induces a scf if the aggregate preferences have top elements.

Social choice function and relation

Example 2

If the space of alternative choices A can be presented in the form

$$A = \prod_{i=1}^{n} A_i$$

where each A_i is controlled by the player i, then the scr can be defined to be

$$\wr\!>\!\int_r \quad = \quad \{\sigma \in A \mid \sigma\ BR\ \sigma\}$$

i.e. the social choices are the equilibria of the game.

Ranking

Definition

A *voting vector* (or a *procedure*) for ℓ candidates is an ℓ-tuple

$$(c_{\ell-1}, c_{\ell-2}, \ldots, c_0)$$

which is descending, i.e. $c_{i+1} \geq c_i$ for all i.

Ranking

- Suppose that there are ℓ candidates in A.
- Let $(c_{\ell-1}, c_{\ell-2}, \ldots, c_0)$ be a voting vector.
- For each i, rename the candidates

$$A = \left\{a_0^{(i)}, a_1^{(i)}, a_2^{(i)}, \ldots a_{\ell-1}^{(i)}\right\}$$

so that

$$a_{\ell-1}^{(i)} \overset{i}{\succ} a_{\ell-2}^{(i)} \overset{i}{\succ} a_{\ell-3}^{(i)} \overset{i}{\succ} \cdots \overset{i}{\succ} a_0^{(i)}$$

and set

$$u_i\left(a_k^{(i)}\right) = c_k$$

Ranking

- Then derive $u : A \to \mathbb{R}$ as

$$u(x) \quad = \quad \sum_{i=1}^{n} u_i(x)$$

and set

$$a \wr{>}\wr_W b \quad \iff \quad u(a) > u(b)$$

Ranking

Instances

- **Borda ranking:** $(\ell - 1, \ell - 2, \ldots, 0)$
- **plurality vote:** $(1, 0, \ldots, 0)$
- **antiplurality vote:** $(1, 1, \ldots, 1, 0)$

Condorcet requirement

Definition

A swf $\wr - \int_w \, : \mathbb{P}^n \to \mathbb{P}$ satisfies the *Condorcet requirement* if

$$a \wr \succ \int_w b \implies \# a\{\succ\}b > \# b\{\succ\}a$$

Borda count violates Condorcet requirement

Example

Consider the preferences

voters	preference
30	$a \succ b \succ c$
1	$a \succ c \succ b$
29	$b \succ a \succ c$
10	$b \succ c \succ a$
10	$c \succ a \succ b$
1	$c \succ b \succ a$

Borda count violates Condorcet requirement

Example

Consider the preferences

voters	preference
30	$a \succ b \succ c$
1	$a \succ c \succ b$
29	$b \succ a \succ c$
10	$b \succ c \succ a$
10	$c \succ a \succ b$
1	$c \succ b \succ a$

Then $b(109) \wr\!\succ\!\int_w a(101) \wr\!\succ\!\int_w c(33)$
but $a(41) \succ b(40)$ and $a(60) \succ c(21)$.

Condorcet ranking

Definition

Given a preference profile $\succ \in \mathbb{P}^n$, the *Condorcet ranking* $\gg$ is defined by setting

$$a \gg b \quad \Longleftrightarrow \quad \#a\{\succ\}b \;>\; \#b\{\succ\}a$$

Condorcet ranking allows cycles

Example

Consider the preferences

voters	preference
23	$a \succ b \succ c$
2	$b \succ a \succ c$
17	$b \succ c \succ a$
10	$c \succ a \succ b$
8	$c \succ b \succ a$

Condorcet ranking allows cycles

Example

Consider the preferences

voters	preference
23	$a \succ b \succ c$
2	$b \succ a \succ c$
17	$b \succ c \succ a$
10	$c \succ a \succ b$
8	$c \succ b \succ a$

Then

$$a(33) \gg b(27) \qquad b(42) \gg c(18) \qquad c(35) \gg a(25)$$

Condorcet ranking allows cycles

Corollary

Condorcet ranking may not be transitive.

Proof

If Condorcet ranking were transitive, then $a \gg b$ and $b \gg c$ and $c \gg a$ would imply $a \gg a$.

But by the definition of Condorcet ranking, this would mean that $\#a\{\succ\}a \; > \; \#a\{\succ\}a$, which is impossible.