Security & Economics — Part 3
Interdependencies of security investments

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Outline

Introduction

Games

Security interdependencies in network economy

Outline

Where are we?

Done: Internal view of security investment

Basic tools for

- evaluating security risks
- comparing costs and benefits benefits
- deciding about the preferred solutions

Where are we?

To do: External view of security interdependencies

How does my neighbor's security
influence my own security investment?

Recall: Preference

Definition

A *preference* over a set A is a binary relation

$$\succ \subseteq A \times A$$

which is

- transitive: $a \succ b \wedge b \succ c \implies a \succ c$
- total: $a \succ b \vee b \succ a \vee a = b$

Recall: Utility

Terminology

A function $u : A \to \mathbb{R}$ is called *utility* when it is used to express a preference relation.

Recall: Utility

Terminology

A function $u : A \to \mathbb{R}$ is called *utility* when it is used to express a preference relation.

Remark

The relation $\succ \subseteq A \times A$ defined

$$a \succ b \quad \Longleftrightarrow \quad u(a) > u(b)$$

is always a preference relation, for any given u.

Recall: Utility

Proposition

Every preference relation can be expressed by many different utility functions.

Utility and value

The word *value*, it is to be observed, has two different meanings, and sometimes expresses the utility of some particular object, and sometimes the power of purchasing other goods which the possession of that object conveys. The one may be called 'value in use ;' the other, 'value in exchange.' The things which have the greatest value in use have frequently little or no value in exchange; and on the contrary, those which have the greatest value in exchange have frequently little or no value in use. Nothing is more useful than water: but it will purchase scarce any thing; scarce any thing can be had in exchange for it. A diamond, on the contrary, has scarce any value in use; but a very great quantity of other goods may frequently be had in exchange for it.

Adam Smith

Utility and value

A valuable property must be

- tranferrable

Utility and value

A valuable property must be

- tranferrable
- scarce

Utility and value

A valuable property must be

- tranferrable
- scarce
- effectively **secured**

Utility and value **require security**

Economics $\subseteq$ Security

- An asset is an asset only if it can be secured.

Security $\subseteq$ Economics

- A protection is effective only if it is cost effective.

Utility paradoxes

"Problems of decision under uncertainty"

- St. Petersburg paradox
- Ellsberg paradox
- Alais paradox

Utility paradoxes

"Problems of decision under uncertainty"

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Homework

Read the Wikipedia articles about these paradoxes. They are fun! Everyone has a different solution. See how you would resolve them!

Reconciling utilities: Games

Definition

A (normal form, von Neumann-Morgenstern) *game* is an n-tuple of utility functions $u = \langle u_i \rangle_{i=1}^{n} : A \to \mathbb{R}^n$ where

- $i = 1, 2, \ldots, n$ are the *players*
- A_i is the set of moves available to the player i
- $A = \prod_{i=1}^{n} A_i$
- $u_i : A \to \mathbb{R}$ is i's utility

From decisions to interactions

- **Decision theory** studies individual preferences:
  - an individual's decides to choose $a \in A$.

From decisions to interactions

- **Decision theory** studies individual preferences:
  - an individual's decides to choose $a \in A$.

- **Game theory** studies the interactions between the individuals with different preferences:
  - players $k = 1, 2, \ldots, n$
  - utilities $u_k : \prod_{i=1}^{n} A_i \to \mathbb{R}$
  - k controls her own moves $a_k \in A_k$
  - k does not control j's choices $a_j \in A_j$ for $j \neq k$

Outline

Introduction

Games

Examples of games

Strategic reasoning

Security interdependencies in network economy

Bimatrix presentation of 2-player games

- $n = 2$
- $A_1 = \{U, D\}$
- $A_2 = \{L, R\}$
- $u = \langle u_1, u_2 \rangle : A_1 \times A_2 \to \mathbb{R}^2$

	L	R
U	$u_1(U,L)$, $u_2(U,L)$	$u_1(U,R)$, $u_2(U,R)$
D	$u_1(D,L)$, $u_2(D,L)$	$u_1(D,R)$, $u_2(D,R)$

Game 1: Prisoners' Dilemma

- $n = 2$
- $A_1 = A_2 = M = \{\text{deny}, \text{confess}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	deny	confess
deny	-1 -1	0 -11
confess	-11 0	-10 -10

Game 1: Prisoners' Dilemma

- $n = 2$
- $A_1 = A_2 = M = \{\text{deny}, \text{confess}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	deny	confess
deny	b / b	a / d
confess	d / a	c / c

$$a \succ b \succ c \succ d$$

Game 2: Arms Race

- $n = 2$
- $A_1 = A_2 = M = \{\text{disarm}, \text{arm}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	disarm	arm
disarm	2 (row), 2 (column)	−1 (row), 3 (column)
arm	3 (row), −1 (column)	1 (row), 1 (column)

Game 2: Arms Race

- $n = 2$
- $A_1 = A_2 = M = \{\text{disarm}, \text{arm}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	disarm	arm
disarm	b, b	d, a
arm	a, d	c, c

$$a > b > c > d$$

Game 2': Arms Race

- $n = 2$
- $A_1 = A_2 = M = \{\text{disarm}, \text{arm}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	disarm	arm
disarm	3 (row), 3 (column)	−1 (row), 2 (column)
arm	2 (row), −1 (column)	1 (row), 1 (column)

Game 3: Stag Hunt

- $n = 2$
- $A_1 = A_2 = M = \{\mathrm{stag}, \mathrm{hare}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	stag	hare
stag	2 2	1 0
hare	0 1	1 1

Game 3: Stag Hunt

- $n = 2$
- $A_1 = A_2 = M = \{\text{stag}, \text{hare}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	stag	hare
stag	a / a	b / c
hare	c / b	b / b

$$a \succ b \succ c$$

Game 4: Chicken in a car

- $n = 2$
- $A_1 = A_2 = M = \{\text{stop}, \text{go}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	stop	go
stop	0 (top right: 0)	−1 (top right: 1)
go	1 (top right: −1)	−10 (top right: −10)

Game 4: Chicken in a car

- $n = 2$
- $A_1 = A_2 = M = \{\text{stop}, \text{go}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	stop	go
stop	b b	a c
go	c a	d d

$$a \succ b \succ c \succ d$$

Game 5: Matching Pennies

- $n = 2$
- $A_1 = A_2 = M = \{\mathrm{H}, \mathrm{T}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	H	T
H	-1 1	1 -1
T	1 -1	-1 1

Game 5: Matching Pennies

- $n = 2$
- $A_1 = A_2 = M = \{\mathrm{H}, \mathrm{T}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	H	T
H	a (bottom left), b (top right)	b (bottom left), a (top right)
T	b (bottom left), a (top right)	a (bottom left), b (top right)

$$a \succ b$$

Game 6: Penalty kick

- $n = 2$
- $A_1 = A_2 = M = \{L, R\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	L	R
L	.58, −.58	.95, −.95
R	.93, −.93	.7, −.7

Game 6: Penalty kick

- $n = 2$
- $A_1 = A_2 = M = \{\mathrm{L}, \mathrm{R}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$

	L	R
H	d, $-d$	a, $-a$
T	b, $-b$	c, $-c$

$$a \succ b \succ c \succ d$$

Notation and terminology

- players: $i = 1, 2, \ldots, n$
- moves: $s_i, t_i \in A_i$
- profiles $s = \langle s_1, \ldots s_n \rangle \in A = \prod_{i=1}^{n} A_i$
- $s_{-k} \in A_{-k} = \prod_{\substack{i=1 \\ i \neq k}}^{n} A_i$

Best response strategy

Definition

A *best response strategy* for a player k in a given game $u : A \to \mathbb{R}^n$ is a relation

$$BR_i \quad \subseteq \quad A_{-k} \times A_k$$

such that

$$a_{-k} \; BR_k \; a_k \quad \iff \quad \forall x_k \in A_k.\ u_k(x_k, a_{-k}) \leq u_k(a_k, a_{-k})$$

Dominant move

Definition

A *dominant move* for a player k in a given game $u : A \to \mathbb{R}^n$ is a move $d_k \in A_k$ which is a best response to all opponent moves. The set of dominant moves for k is thus

$$\mathrm{Dmn}_k \quad = \quad \{d_k \mid \forall x_{-k}.x_{-k}\ BR_k d_k\}$$

i.e.

$$d_k \in \mathrm{Dmn}_k \quad \iff \quad \forall x \in A.\ u_k(x_k, x_{-k}) \leq u_k(d_k, x_{-k})$$

Dominant move equilibrium

Definition

A *dominant move equilibrium* in a given game $u : A \to \mathbb{R}^n$ is a profile $d \in A$ which consists of dominant moves. The set of dominant move equilibria is thus

$$\mathrm{Dmn} = \prod_{i=1}^{n} \mathrm{Dmn}_i$$

i.e.

$$d \in \mathrm{Dmn} \iff \forall i \le n \forall x \in A.\ u_i(x) \le u_i(d_i, x_{-i})$$

Dominant move equilibrium

Exercise

Explore which of the 7 games have dominant move equilibria.

Best response profile

Definition

A *best response profile* for a given game $u : A \to \mathbb{R}^n$, where $A = \prod_{i=1}^{n} A_i$ is a relation

$$BR \quad \subseteq \quad A \times A$$

such that

$$s\ BR\ t \quad \iff \quad \forall k.\ s_{-k}\ BR_k\ t_k$$

Nash equilibrium

Definition

A *(Nash) equilibrium* for a given game $u : A \to \mathbb{R}^n$, where $A = \prod_{i=1}^{n} A_i$ is a profile $s \in A$ such that

$$s \; BR \; s$$

Nash equilibrium

Exercise

Explore which of the 7 games have Nash equilibria.

Nash equilibrium

Proposition

Every dominant equlibrium is a Nash equilibrium.

Mixed moves

Definition

A *mixed move* α for a player k is a convex combination of moves from A_k, i.e.

$$\alpha = \sum_{j=1}^{m} \alpha_j \cdot a_k^j$$

where $\sum_{j=1}^{n} \alpha_j = 1$ and $a_k^1, a_k^2, \ldots a_k^m \in A_k$.

Mixed moves

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The set of mixed moves over A_k is thus

$$\Delta A_k \cong \coprod_{m=1}^{\infty} \left\{ \alpha \in \mathbb{R}^m \mid \sum_j \alpha^j = 1 \right\} \times A_k^m$$

Mixed moves

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The set of mixed moves over A_k is thus

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The unmixed moves from A_k are called *pure*.

Mixed moves

Remark 1

A mixed move α for a player k can equivalently be viewed as a finitely supported probability distribution $\alpha : A_k \to [0, 1]$, i.e. satisfying

$$\sum_{x \in A_k} \alpha(x) = 1 \qquad \#\{x \in A_k \mid \alpha(x) \neq 0\} < \infty$$

Mixed moves

Remark 2

Utility functions and the notion of (normal form) game extend to mixed moves:

$$\frac{u : \prod_{i=1}^{n} A_i \to \mathbb{R}^n}{\widehat{u} : \prod_{i=1}^{n} \Delta A_i \to \mathbb{R}^n}$$

by setting

$$\widehat{u}_i(\ldots \alpha_k \ldots) \quad = \quad \sum_{j=1}^{m} \alpha_k^j \cdot u_i\left(a_k^j\right)$$

Nash's Theorem

Theorem (Nash)

The Nash equilibrium in mixed moves exists for every game between finitely many players, with finitely many pure moves.

Hawk and Dove with parameters

- $n = 2$
- $A_1 = A_2 = M = \{\text{retreat}, \text{attack}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$
- $w =$ winnings to be shared
- $c =$ cost of battle

	retreat	attack
retreat	$\frac{w}{2}$, $\frac{w}{2}$	0, w
attack	w, 0	$\frac{w}{2} - c$, $\frac{w}{2} - c$

Hawk and Dove with parameters

- if $0 < w$ and $c < \frac{w}{2}$, then
  - the dominant equilibrium is $\langle \textit{attack}, \textit{attack} \rangle$
- if $0 < w$ and $c > \frac{w}{2}$, then
  - there is no dominant equilibrium
  - $\langle \textit{attack}, \textit{retreat} \rangle$ and $\langle \textit{retreat}, \textit{attack} \rangle$ are Nash equilibria

Hawk and Dove with parameters

- if $0 > w$ and $c > \frac{w}{2}$, then
  - the dominant equilibrium is $\langle \mathit{retreat}, \mathit{retreat} \rangle$
- if $0 > w$ and $c < \frac{w}{2}$, then
  - there is no dominant equilibrium
  - $\langle \mathit{attack}, \mathit{attack} \rangle$ and $\langle \mathit{retreat}, \mathit{retreat} \rangle$ are Nash equilibria

Outline

Security Investment Game

- $n = 2$
- $A_1 = A_2 = M = \{\text{invest}, \text{don't}\}$
- $u = \langle u_1, u_2 \rangle : M^2 \to \mathbb{R}^2$
- C = cost of the investment
- L = value under threat
- v = vulnerability: probability of successful attack
- w = total transferred vulnerability
  - received from the neighbors

Security Investment Game

	invest	don't
invest	$-C$ $-C$	$-vL$ $-C-wL$
don't	$-C-wL$ $-vL$	$-vL-(1-v)wL$ $-vL-(1-v)wL$

Security Investment Game

- if $C < v(1 - w)L$ then
  - $\langle$invest, invest$\rangle$ is dominant equilibrium
- if $v(1 - w)L < C < vL$ then
  - there is no dominant equilibrium
  - $\langle$invest, invest$\rangle$ and $\langle$don't, don't$\rangle$ are Nash equilibria
- if $vL < C$ then
  - $\langle$don't, don't$\rangle$ is dominant equilibrium