Security & Economics — Part 5

Market with intermediaries and advertising

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Outline

Introduction

Sponsored search

Market with intermediaries

Outline

Introduction

Market is a system of exchange protocols

- compute the prices
- regulate the exchange

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- compute the prices
- regulate the exchange

We focus on computing the prices.

Introduction

An auction is a market organized by

- a seller: supply auction
- a buyer: procurement auction

Introduction

Markets in general are organized by

- **universal buyers/sellers**
  - merchants, traders, dealers,
  - entrepreneurs,
  - advertisers (push), solicitors (pull)

who mediate among the buyers and the sellers

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Markets in general are organized by

- **universal buyers/sellers**
  - merchants, traders, dealers,
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  who mediate among the buyers and the sellers

- just like the **universal goods**
  - money
  - securities (bonds, equity, derivatives)

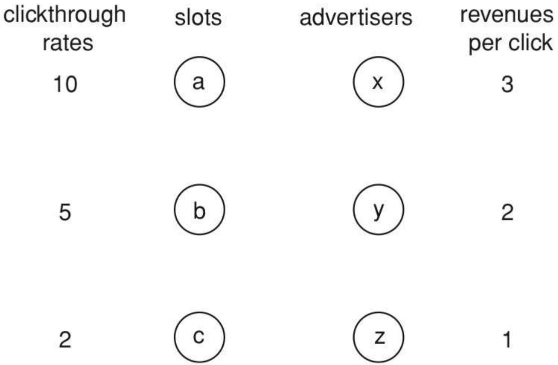
  mediate among the goods

In this lecture

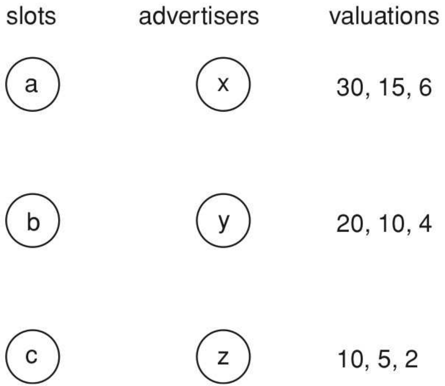
- Multi-item auctions
  - example: sponsored search
  - problem of incentive compatibility
    - Later: *What is the value of advertising?*
- Market with intermediaries
  - traders' strategies
  - trading profits and social benefits

Outline

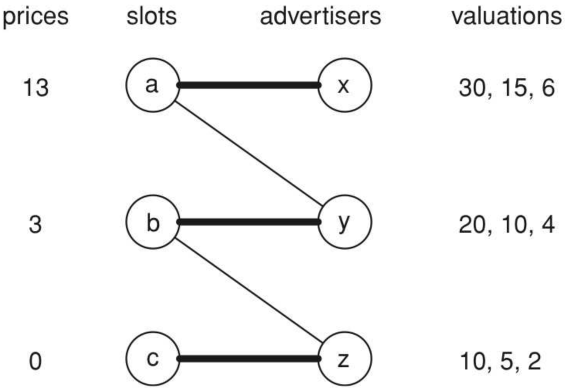
Sponsored search setting



Sponsored search as a matching problem



Sponsored search as a market



Market mechanism

- n buyers, n item
  - take $n = \{0, 1, \ldots, n-1\}$

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- i's utility $u_i \in \mathbb{R}$ is

$$u_i = v_{i\sigma_{vp}(i)} - p_{\sigma_{vp}(i)}$$

Goal of the market mechanism

Maximize social welfare, i.e. buyers' total payoff

$$\begin{aligned} U(v,p) &= \sum_{i \in n} u_i \\ &= \sum_{i \in n} v_{i\sigma_{vp}(i)} - p_{\sigma_{vp}(i)} \\ &= \sum_{i \in n} v_{i\sigma(i,v)} - P \end{aligned}$$

where $P = \sum_{i<n} p_i$

Markets respect preference

To maximize utility, $\sigma_{vp} : n \to n$ maximizes valuations

$$v_{i\sigma(i,v)} \quad \geq \quad v_{ij}$$

Position auction mechanism

- n bidders, n positions

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- price per position $\pi_{ij}(b) = p_i(b) \cdot r_j$ where
  - price per click $p(b) = \big(p_i(b)\big)_n$

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- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) \;=\; v_{i\tau(i,b)} - \pi_{i\tau(i,b)}(b) \;=\; \big(w_i - p_i(b)\big) \cdot r_{\tau(i,b)}$$

Goal of the position auction mechanism

Maximize seller's revenue

$$P(b) = \sum_{i<n} \pi_{i\tau(i,b)}(b) = \sum_{i<n} p_i(b) \cdot r_{\tau(i,b)}$$

where

- all p_i grow with b
- bidder i bids b_i to maximize $u_i(b)$.

Position auctions respect preference

To maximize $p_i(b)$ with $u_i(b)$ always use

- $\tau(i, b) < \tau(j, b) \implies b_i \geq b_j$, i.e.
- $\tau(i, b) = j$ if b_i is j-th largest entry in b

Assumption

- The bidders are ordered[1] by their bids

$$b_1 > b_2 > b_3 > \cdots > b_n$$

- The positions are ordered by click-through rates

$$r_1 > r_2 > r_3 > \cdots > r_n$$

[1] Recall: Since the priority of equal bids can be resolved by ordering the bidders e.g. by their names, with no loss of generality we assume that there are no equal bids.

Generalized Second Price Auction

- n bidders, n positions
- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
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- price per click $p_i(b) = b_{i+1}$

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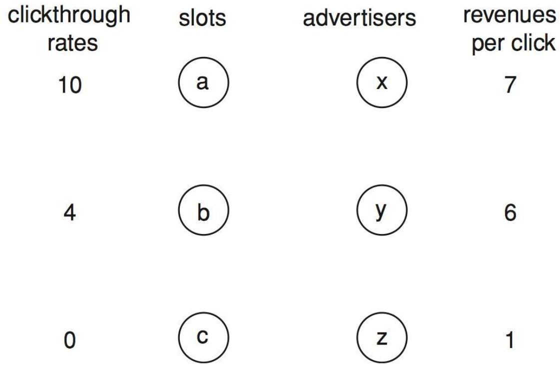
- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) \ = \ \left(w_i - b_{i+1}\right) \cdot r_i$$

Does GSP encourage truthful bidding?



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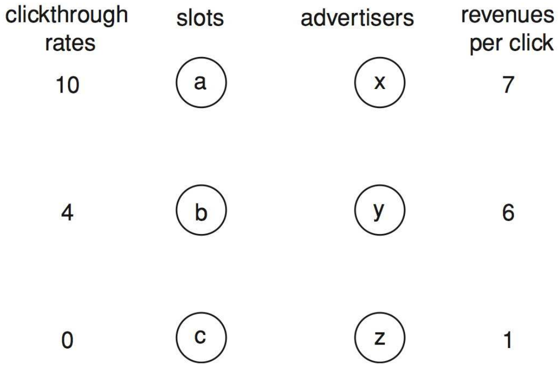
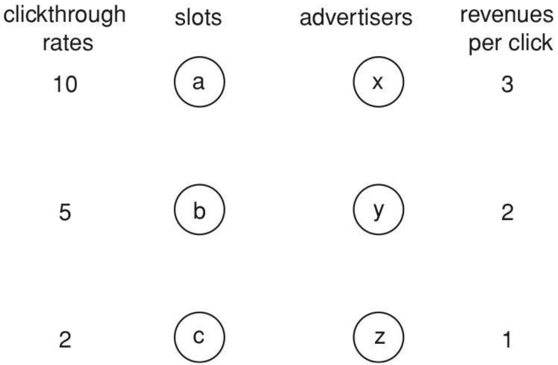
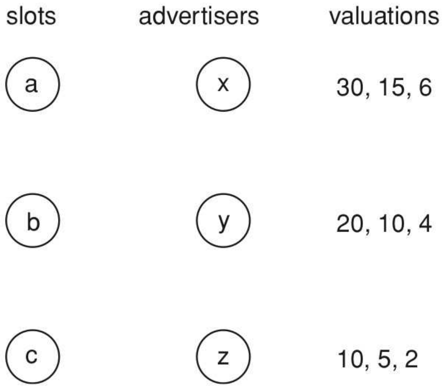


- with truthful bid: $u_x(7, 6, 1) = (7 - 6) \cdot 10 = 10$
- with untruthful bid: $u_x(5, 6, 1) = (7 - 1) \cdot 4 = 24$

Position auction example

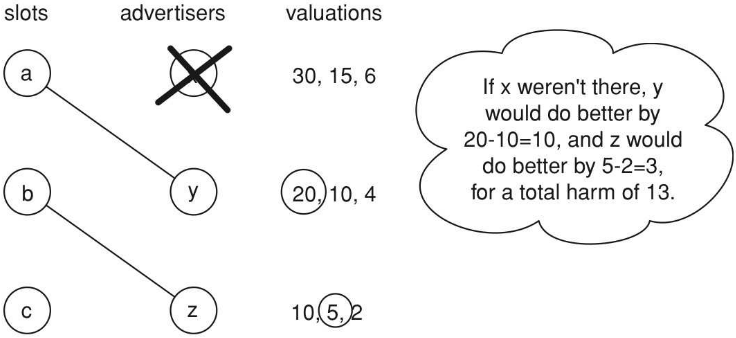


Matching problem view



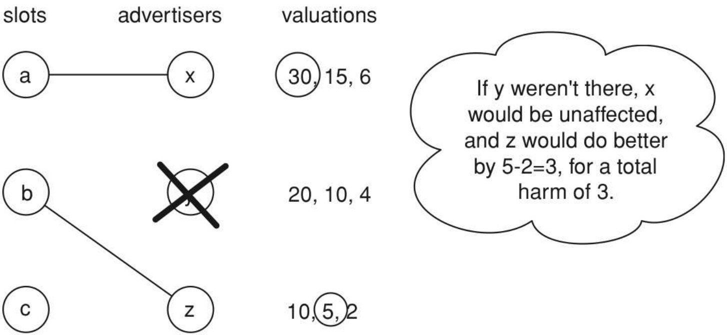
Idea

- How much does *x* subtract from social welfare?



Idea

- How much does *y* subtract from social welfare?



Idea: Vickrey, Clarke, Groves

- Each bidder should pay the cost that their bid incurs on social welfare
  - i.e., the sum of the losses that they cause to other bidders

Vickrey-Clarke-Groves Auction

Notation

- B — set of bidders
- S — set of sellers (items)
- $v = (v_{ij})_{B \times S}$ — bidders' valuations
- V_B^S — maximal total valuation

Vickrey-Clarke-Groves Auction

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- B — set of bidders
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- V_B^S — maximal total valuation

Remark

- If $\#B < \#S$, then add $\#S - \#B$ bidders with all valuations 0
- If $\#B > \#S$, then add $\#B - \#S$ sellers valued 0 by all.

Vickrey-Clarke-Groves Auction

Remember the assumption

- The bidders are ordered by their bids

$$b_1 > b_2 > b_3 > \cdots > b_n$$

- The positions are ordered by click-through rates

$$r_1 \geq r_2 \geq r_3 \geq \cdots \geq r_n$$

Vickrey-Clarke-Groves Auction

- n bidders, n positions

- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
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- price per item $\pi_{ij}(b) = V^{S}_{B\setminus i} - V^{S\setminus j}_{B\setminus i}$

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- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) = v_{ii} - \pi_{ii}(b)$$

Vickrey-Clarke-Groves Auction

Theorem

The VCG auction is incentive compatible: truthful bidding is the unique Nash equilibrium for all players.

Vickrey-Clarke-Groves Auction

Corollary

The VCG auction maximizes social wellfare, i.e. the total utility of bidders.

Problem

Homework

For the sponsored search market

clickthrough rates	slots	advertisers	revenues per click
10	a	x	7
4	b	y	6
0	c	z	1

compute seller's revenue (i.e. the total of the prices charged for all items) if the positions are auctioned by a GSP auction and by a VCG auction

Show that neither of these mechanisms maximizes seller's revenue.

Billion $ problem

Design an auction mechanism that maximizes seller's revenue.

Outline

Toy market

- There is just one type of goods.
- Every buyer needs to buy one item.
- Every seller needs to sell one item.

Toy market

- buyers $\mathcal{B} = \{B_1, B_2, \ldots B_n\}$ have valuations v_i
- sellers $\mathcal{S} = \{S_1, S_2, \ldots S_n\}$ have valuations w_j

Toy market

- buyers $\mathcal{B} = \{B_1, B_2, \ldots B_n\}$ have valuations v_i
- sellers $\mathcal{S} = \{S_1, S_2, \ldots S_n\}$ have valuations w_j

Remark

If the numbers are different, then add

- buyers with the valuation 0, or
- sellers with the valuation 1.

Toy market

Goal of the market

Find a bijection $\sigma : \mathcal{B} \to \mathcal{S}$ that maximizes social benefit

$$SB_\sigma = \sum_{i=1}^{n} v_i - w_{\sigma i}$$

Market with intermediaries

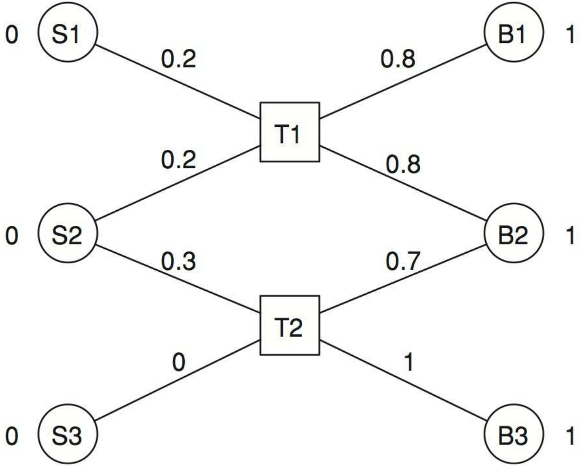
- Just like the goods are compared through **universal goods**
  - money, securities
- the buyers' and the sellers' are connected through **universal buyers/sellers**
  - merchants, traders, advertisers

Market with intermediaries

The intermediaries mediate the flows

- merchants buy, move and sell goods
- traders buy and sell goods without moving them
- advertisers and solicitors move information

Market with intermediaries



Market with intermediaries as a game

- buyers $\mathcal{B} = \{B_1, B_2, B_3\}$
  - their reserve prices (valuations) $v_1 = v_2 = v_3 = 1$
- sellers $\mathcal{S} = \{S_1, S_2, S_3\}$
  - their reserve price (valuations) $w_1 = w_2 = w_3 = 0$
- traders $\mathcal{T} = \{T_1, T_2\}$
  - ask relation $T_1 \stackrel{a}{\to} B_1$, $T_1 \stackrel{a}{\to} B_2$, $T_2 \stackrel{a}{\to} B_2$, $T_2 \stackrel{a}{\to} B_3$
    - T_1's buyers $\mathcal{B}_1 = \{B_1, B_2\}$
    - T_2's buyers $\mathcal{B}_2 = \{B_2, B_3\}$
  - bid relation $S_1 \stackrel{b}{\to} T_1$, $S_2 \stackrel{b}{\to} T1$, $S_2 \stackrel{b}{\to} T_2$, $S_3 \stackrel{b}{\to} T_2$
    - T_1's sellers $\mathcal{S}_1 = \{S_1, S_2\}$
    - T_2's sellers $\mathcal{S}_2 = \{S_2, S_3\}$

Market with intermediaries as a game

Setting

- buyers $\mathcal{B} = \{B_1, \ldots, B_n\}$
  - B_i's reserve price (valuation) is v_i
- sellers $\mathcal{S} = \{S_1, \ldots, S_n\}$
  - S_j's reserve price (valuation) is w_j
- traders $\mathcal{T} = \{T_1, \ldots T_m\}$
  - ask relation $\stackrel{a}{\to} \subseteq \mathcal{T} \times \mathcal{B}$
    - T_k's buyers $\mathcal{B}_k = \{B_i \in \mathcal{B} \mid T_k \stackrel{a}{\to} B_i\}$
  - bid relation $\stackrel{b}{\to} \subseteq \mathcal{S} \times \mathcal{T}$
    - T_k's sellers $\mathcal{S}_k = \{S_j \in \mathcal{S} \mid S_j \stackrel{b}{\to} T_k\}$

Market with intermediaries as a game

Game

players: traders $T_1, \ldots, T_m$

moves: for the trader T_k's the set of moves is

$$\begin{aligned} P_k &= Pb_k \times Pa_k, \quad \text{where} \\ Pb_k &= \mathbb{R}^p \text{ with } p = \#\mathcal{S}_k \\ Pa_k &= \mathbb{R}^q \text{ with } q = \#\mathcal{B}_k \end{aligned}$$

where

- $b_k = \langle b_{k1}, b_{k2}, \ldots, b_{kp} \rangle \in Pb_k$ are T_k's bid prices for all $S_j \in \mathcal{S}_k$
- $a_k = \langle a_{k1}, a_{k2}, \ldots, a_{kq} \rangle \in Pa_k$ are T_k's ask prices for all $B_i \in \mathcal{B}_k$

Market with intermediaries as a game

Play

- Each T_k announces its bid and ask prices $p_k = \langle b_k, a_k \rangle$
- Each S_j agrees to sell to a T_k with a maximal b_{kj}
- Each B_i agrees to buy from a T_k with a minimal a_{ki}
- Each T_k thus forms the sets of
  - suppliers $\mathcal{MS}_k = \left\{S_j \in \mathcal{S}_k \mid \forall \ell.\ b_{\ell j} \leq b_{kj}\right\}$
  - customers $\mathcal{MB}_k = \{B_i \in \mathcal{B}_k \mid \forall \ell.\ a_{ki} \leq a_{\ell i}\}$

Market with intermediaries as a game

Trader T_k's utility

- If $\#\mathcal{MB}_k \leq \#\mathcal{MS}_k$ (sufficient supplies) then

$$u_k\left(\vec{p}\right) = \sum_{B_i \in \mathcal{MB}_k} a_{ki} - \sum_{S_j \in \mathcal{MS}_k} b_{kj}$$

- If $\#\mathcal{MB}_k > \#\mathcal{MS}_k$ (insufficient supplies) then

$$u_k\left(\vec{p}\right) = \sum_{B_i \in \mathcal{MB}_k^+} a_{ki} - \sum_{S_j \in \mathcal{MS}_k} b_{kj} - \sum_{B_i \in \mathcal{MB}_k^-} a_{ki}$$

  where $\mathcal{MB}_k = \mathcal{MB}_k^+ \cup \mathcal{MB}_k^-$, and
  - $\mathcal{MB}_k^+$ is the set of $\#\mathcal{MS}_k$ buyers who accepted the highest ask prices
  - $\mathcal{MB}_k^-$ are the remaining $\#\mathcal{MB}_k - \#\mathcal{MS}_k$ buyers with the lowest ask prices

Distribution of social benefit

If the bijection $\sigma : \mathcal{B} \to \mathcal{S}$ that maximizes social benefit

$$SB_\sigma = \sum_{i=1}^{n} v_i - w_{\sigma i}$$

is found through the traders $\kappa : \mathcal{B} \to \mathcal{T}$, then the benefit is distributed

$$SB_\sigma = \sum_{i=1}^{n} \underbrace{\left(v_i - a_{\kappa(i)i}\right)}_{UB} + \underbrace{\left(a_{\kappa(i)i} - b_{\kappa(i)\sigma(i)}\right)}_{UT} + \underbrace{\left(b_{\kappa(i)\sigma(i)} - w_{\sigma i}\right)}_{US}$$

where

- *UB* is the utility of the buyer
- *UT* is the utility of the trader
- *US* is the utility of the seller

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where

- *UB* is the utility of the buyer
- *UT* is the utility of the trader
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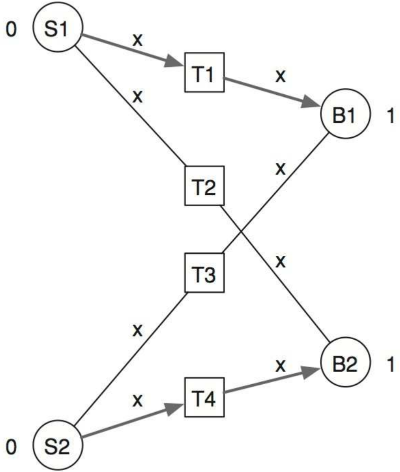
The traders maximize *UT*.

Distribution of social benefit

- But how do the traders achieve their payoffs?
- What are the equilibria in the trading game?

Implicit perfect competition



Indifference principle

At equilibrium

- All bid prices offered to a seller must be equal
- The seller will accept the bid from the trader who has access to the highest paying buyers
  - because that trader can increase the bid by ε

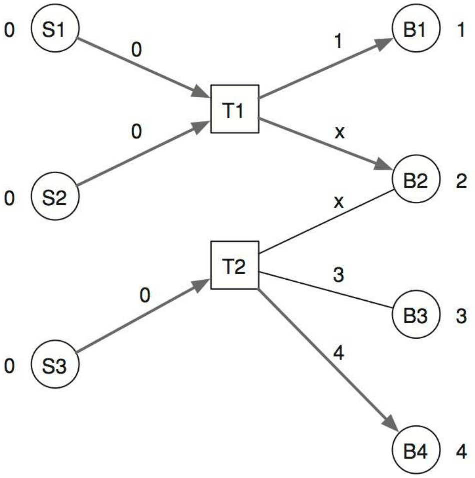
Indifference principle

At equilibrium

- All bid prices offered to a seller must be equal
- The seller will accept the bid from the trader who has access to the highest paying buyers
  - because that trader can increase the bid by ε

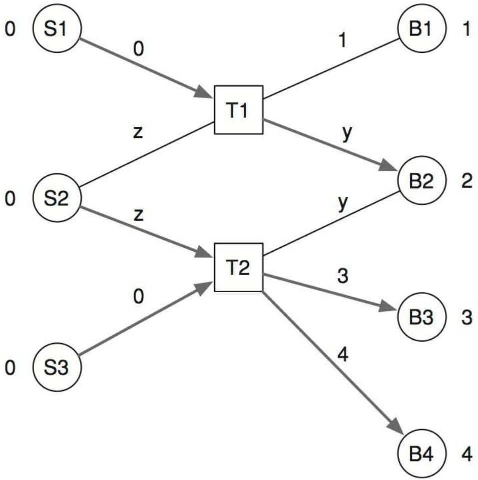
- All ask prices offered to a buyer must be equal
- The buyer will accept the offer from the trader who has access to the lowest charging sellers
  - because that trader can undercut the offer by ε

Ripple effects



$$0 \leq x \leq 2$$

Ripple effects



$$1 \leq y \leq 2 \qquad 1 \leq z \leq 3$$