Security & Economics — Part 6
Network effects and self-fulfilling claims

Dusko Pavlovic

Spring 2014

Outline

Introduction

Positive network effects and self-fulfilling expectations

Negative network effects and minority game

Outline

Introduction

Positive network effects and self-fulfilling expectations

Negative network effects and minority game

Three witches

The Tragedy of Macbeth

Three witches' prophecy

First Witch: All hail, Macbeth! Hail to thee, Thane of Glamis!

Second Witch: All hail, Macbeth, hail to thee, Thane of Cawdor!

Third Witch: All hail, Macbeth, thou shalt be King hereafter!

The Tragedy of Macbeth

Self-fulfilling prophecy

1. Macbeth is just a little spooked that the witches knew that he was Thane of Glamis.

The Tragedy of Macbeth

Self-fulfilling prophecy

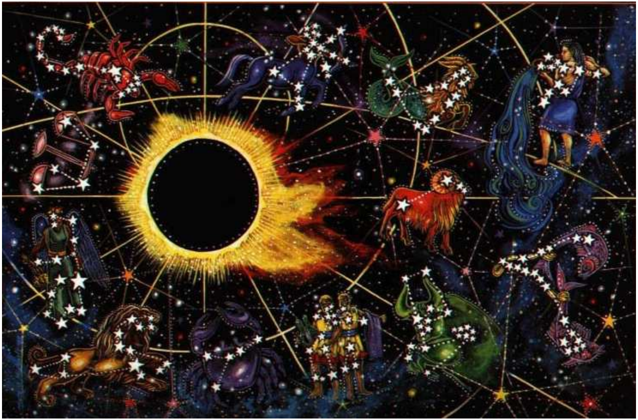
1. Macbeth is just a little spooked that the witches knew that he was Thane of Glamis.
2. Macbeth gets promoted into Thane of Cawdor by the King — and recognizes the prophecy.

The Tragedy of Macbeth

Self-fulfilling prophecy

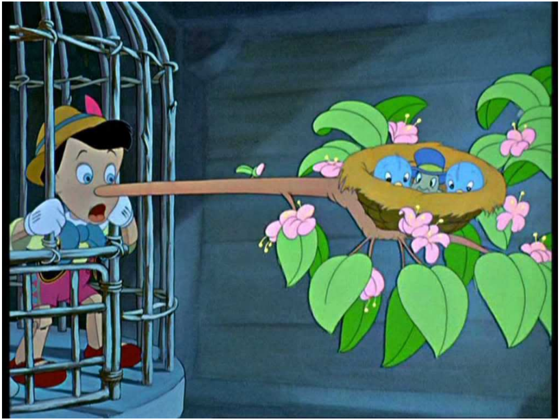
1. Macbeth is just a little spooked that the witches knew that he was Thane of Glamis.
2. Macbeth gets promoted into Thane of Cawdor by the King — and recognizes the prophecy.
3. Macbeth kills the King — and realizes the prophecy.

How does future forecasting work?



Why do we believe in stars at 30000 light years away?

Is lying sometimes a rational strategy?



Is lying effective? If not, why do we lie?

Why do we advertise?

If the market is efficient, and computes the right prices, why is it rational to invest in advertising?

Outline

Introduction

Positive network effects and self-fulfilling expectations

Economy of demand and intrinsic values

Economy with externalities

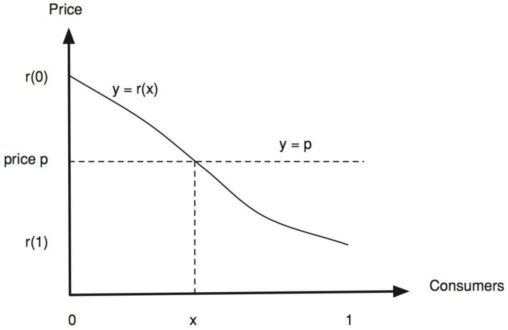
Negative network effects and minority game

Demand and valuation

Market computes the demand for a product

demand: $q(y) = x$ — the quantity required at the price y

valuation: $r(x) = y$ — the reserve price for x consumers

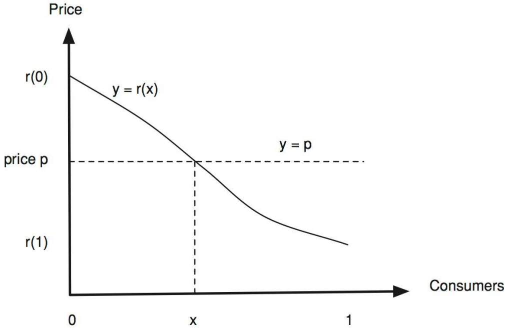


Demand and valuation are inverses

Market computes the demand for a product

demand: $q(r(x)) = x \in [0,1]$ — fraction of consumers

valuation: $r(q(y)) = y \in [0,\infty]$ — value derived from use



Intuitions

demand: consumers' names are $x \in [0,1]$

- ordered by their valuations for the good Γ
- if x purchases Γ, then
  - all $x' \in [0,x]$ purchase Γ,
  - because $r(x') \geq r(x)$, and

Intuitions

demand: consumers' names are $x \in [0, 1]$

- ordered by their valuations for the good Γ
- if x purchases Γ, then
  - all $x' \in [0, x]$ purchase Γ,
  - because $r(x') \geq r(x)$, and

valuation: prices are $y \in [0, \infty]$

- ordered by the demand for Γ
- if $y > y'$ then
  - $q(y) < q(y')$, and
  - all $x \in [0, q(y')]$ will buy Γ
  - for $r(x) \in [y', 1]$

Equilibrium of demand and supply

- Let $p = y^*$ be the fixed (average) production cost.
- The products will be priced at $y > y^*$.
- The buyers $x < x^* = q(y^*)$ will purchase Γ at
- the prices $y > y^* = r(x^*)$.
- The market will demand $x^* = q(y^*)$ of Γ.

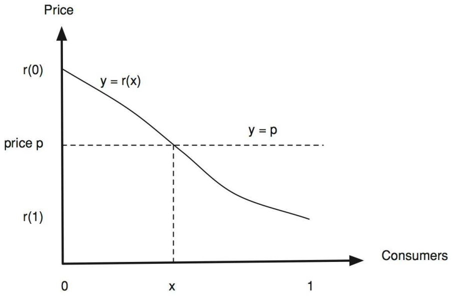
Equilibrium of demand and supply

- Let $p = y^*$ be the fixed (average) production cost.
- The products will be priced at $y > y^*$.
- The buyers $x < x^* = q(y^*)$ will purchase Γ at
- the prices $y > y^* = r(x^*)$.
- The market will demand $x^* = q(y^*)$ of Γ.
- $\langle x^*, y^* \rangle$ is the *demand-supply equilibrium*
  - where $y^* = r(x^*)$

Social benefit at the equilibrium

$$SB(x^*) = \int_0^{x^*} r(x)dx - x^* r(x^*)$$

is the difference of the total utility $\int_0^{x^*} r(x)dx$ and the production cost $x^* p = x^* r(x^*)$, i.e. the upper triangle in



Intrinsic values and externalities

Intrinsic values of goods are expressed through their market prices and their production costs.

Externalities are the values of goods taken by those who are neither producers nor consumers of these goods.

Examples of externalities

Positive:
- public health, security, education
- freeware, creative commons
- social adoption of shared applications

Negative:
- pollution, environmental change
- exploitation of resources (e.g. fishing)
- systemic risk (e.g. in banking)
- congestion
- price increase due to demand

Valuations with externalities

Market adoption influences the valuation

$$v(x,z) \quad = \quad r(x) \cdot f(z)$$

where

- $r(x)$ is the *intrinsic valuation*
  - x's reserve price if market fully adopts Γ
- $f(z)$ is the *network effect*
  - price change if z-part of the market adopts Γ

Valuations with positive externalities

- $r : [0,1] \to [0,1]$ is monotone decreasing function
  - e.g. $r(x) = 1 - x$
    - $r(0) = 1$: Γ is not valued at ∞^* by anyone
    - $r(1) = 0$: Γ has no value for some consumers
- $f : [0,1] \to [0,1]$ is monotone increasing function
  - e.g. $f(z) = z$
    - $f(0) = 0$: Γ has no value if no adoption
    - $f(1) = 1$: Γ has full value with full adoption

*[0, 1] represents the price interval $[0, \infty]$.

Network adoption equilibrium

- Let p^* be the fixed (average) production cost.

Network adoption equilibrium

- Let p^* be the fixed (average) production cost.

- Suppose that x **knows** that
  - z^*-part of the market has adopted Γ

Network adoption equilibrium

- Let p^* be the fixed (average) production cost.

- Suppose that x **knows** that
  - z^*-part of the market has adopted Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff x'$ has bought Γ

Network adoption equilibrium

- Let p^* be the fixed (average) production cost.

- Suppose that x **knows** that
  - z^*-part of the market has adopted Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff x'$ has bought Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff r(x')f(z^*) \geq p^*$

Network adoption equilibrium

- Let p^* be the fixed (average) production cost.

- Suppose that x **knows** that
  - z^*-part of the market has adopted Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff x'$ has bought Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff r(x')f(z^*) \geq p^*$

    $\Downarrow$
  - $r(x)f(z^*) \geq p^* \iff x \in [0, z^*]$

Network adoption equilibrium

- Let p^* be the fixed (average) production cost.
- Suppose that x **knows** that
  - z^*-part of the market has adopted Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff x'$ has bought Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff r(x')f(z^*) \geq p^*$

    $\Downarrow$
  - $r(x)f(z^*) \geq p^* \iff x \in [0, z^*]$

    $\Updownarrow$
  - x will buy $\Gamma \iff x \leq z^*$

Network adoption equilibrium

- Let p^* be the fixed (average) production cost.
- Suppose that x **knows** that
  - z^*-part of the market has adopted Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff x'$ has bought Γ

    $\Updownarrow$
  - for all x' holds $x' \in [0, z^*] \iff r(x')f(z^*) \geq p^*$

    $\Downarrow$
  - $r(x)f(z^*) \geq p^* \iff x \in [0, z^*]$

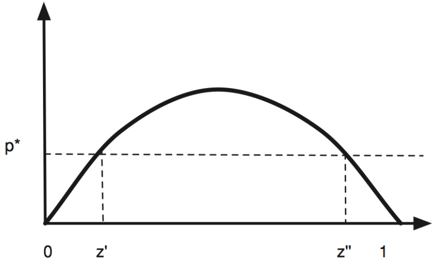
    $\Updownarrow$
  - x will buy $\Gamma \iff x \leq z^*$
- $\langle z^*, p^* \rangle$ is the *network adoption equilibrium*
  - where $p^* = r(z^*)f(z^*)$

Calculating equilibria

Given

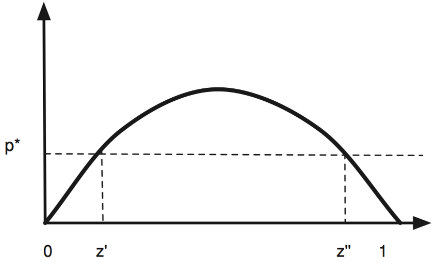
- fixed production price p^*
- reserved price function $r(z) = 1 - z$
- network effect $f(z) = z$
- valuation $v(z) = z(1-z) = z - z^2$

the equilibria $\langle \hat{z}, p^* \rangle$ satisfy $\hat{z} - \hat{z}^2 = p^*$.



Dynamics of market adoption

- $z \in [0, z')$: $v(z) < p^*$ causes $z \searrow 0$
- $z = z'$: $v(z) = p^*$ makes z stable
- $z \in (z', z'')$: $v(z) > p^*$ causes $z \nearrow z''$
- $z = z''$: $v(z) = p^*$ makes z stable
- $z \in (z'', 1]$: $v(z) < p^*$ causes $z \searrow z''$



Tipping point

The Secret of Network Startups

The unstable equilibrium z' is a *tipping point*:

- If the adoption is not pushed to z', the demand will drop to 0.
- If the adoption is pushed past z', the demand will grow to z''.

Tipping point

The Silicon Valley Imperative (Brian Arthur)

- Push down z':
  - lower the price p^* (free trials . . .)
  - widen the parabola $v(z)$ by speeding up $f(z)$

Tipping point

The Silicon Valley Imperative (Brian Arthur)

- Push down z':
  - lower the price p^* (free trials . . .)
  - widen the parabola $v(z)$ by speeding up $f(z)$

$$\Downarrow$$

- The adoption attractor z'' will go up.

Self-fulfilling expectation equilibrium

- Let p^* be the fixed (average) production cost.

Self-fulfilling expectation equilibrium

- Let p^* be the fixed (average) production cost.
- Suppose that x **believes** that z-part of the market has adopted Γ (which may not be true).

Self-fulfilling expectation equilibrium

- Let p^* be the fixed (average) production cost.
- Suppose that x **believes** that z-part of the market has adopted Γ (which may not be true).
  - x purchases $\Gamma \iff r(x)f(z) \geq p^*$

Self-fulfilling expectation equilibrium

- Let p^* be the fixed (average) production cost.
- Suppose that x **believes** that z-part of the market has adopted Γ (which may not be true).
  - x purchases $\Gamma \iff r(x)f(z) \geq p^*$

    $\Updownarrow$
  - x purchases $\Gamma \iff x \leq r^{-1}\left(\frac{p^*}{f(z)}\right)$

Self-fulfilling expectation equilibrium

- Let p^* be the fixed (average) production cost.
- Suppose that x **believes** that z-part of the market has adopted Γ (which may not be true).
  - x purchases $\Gamma \iff r(x)f(z) \geq p^*$

    $\Updownarrow$
  - x purchases $\Gamma \iff x \leq r^{-1}\left(\frac{p^*}{f(z)}\right)$
- The true market adoption (depending on the belief z) is

$$g(z) = q\left(\frac{p^*}{f(z)}\right)$$

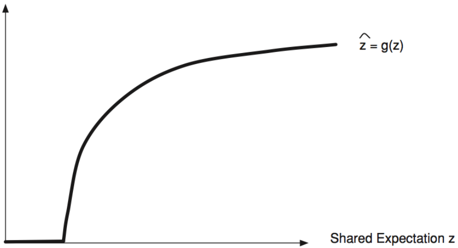
because $r^{-1} = q$.

Example of adoption function

Given

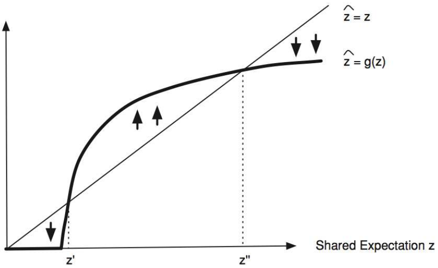
- fixed production price p^*
- reserved price $r(z) = 1 - z$, demand $q(z) = r^{-1}(z) = 1 - z$
- network effect $f(z) = z$

the true adoption is $\widehat{z} = g(z) \;=\; \begin{cases} 0 & \text{if } z \leq p^* \\ 1 - \frac{p^*}{z} & \text{otherwise} \end{cases}$



Finding self-fulfilling equilibrium

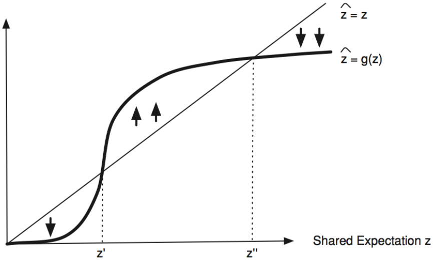
- $g(z) = \widehat{z} \le z \in [0, z')$: $v(\widehat{z}) < p^*$ causes $\widehat{z} \searrow 0$
- $g(z) = \widehat{z} = z'$: $v(\widehat{z}) = p^*$ makes $\widehat{z}$ stable
- $g(z) = \widehat{z} \ge z \in (z', z'')$: $v(\widehat{z}) > p^*$ causes $\widehat{z} \nearrow z''$
- $g(z) = \widehat{z} = z''$: $v(\widehat{z}) = p^*$ makes $\widehat{z}$ stable
- $g(z) = \widehat{z} \le z \in (z'', 1]$: $v(\widehat{z}) < p^*$ causes $\widehat{z} \searrow z''$



for $g(z)$ as in the example

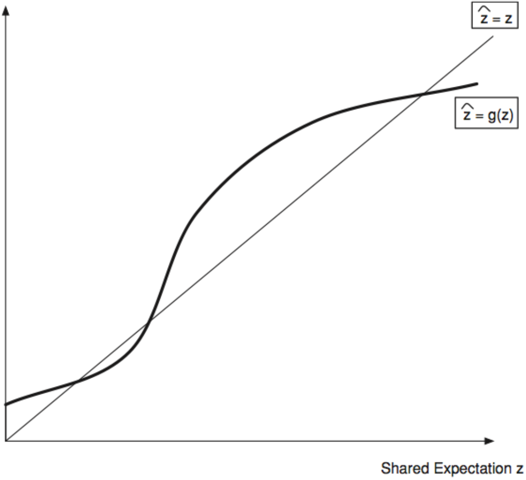
Finding self-fulfilling equilibrium

- $g(z) = \widehat{z} \leq z \in [0, z')$: $v(\widehat{z}) < p^*$ causes $\widehat{z} \searrow 0$
- $g(z) = \widehat{z} = z'$: $v(\widehat{z}) = p^*$ makes $\widehat{z}$ stable
- $g(z) = \widehat{z} \geq z \in (z', z'')$: $v(\widehat{z}) > p^*$ causes $\widehat{z} \nearrow z''$
- $g(z) = \widehat{z} = z''$: $v(\widehat{z}) = p^*$ makes $\widehat{z}$ stable
- $g(z) = \widehat{z} \leq z \in (z'', 1]$: $v(\widehat{z}) < p^*$ causes $\widehat{z} \searrow z''$

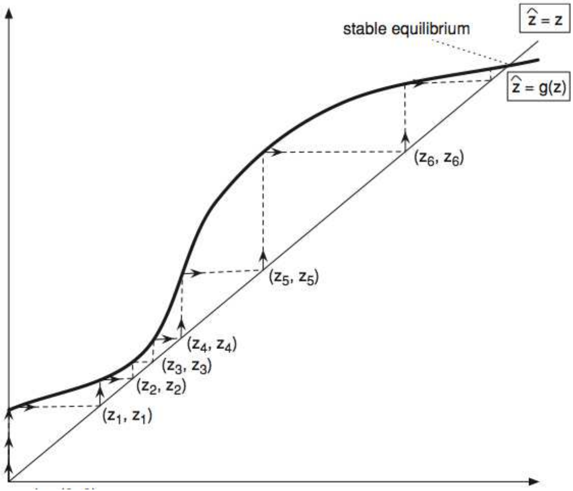


for general $g(z)$

Self-fulfilling equilibrium when $f(0) > 0$



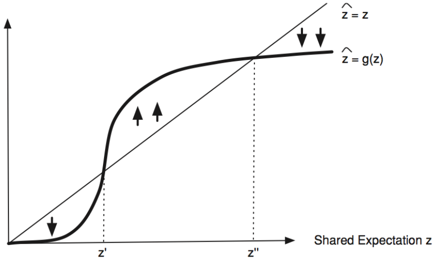
Self-fulfilling equilibrium when $f(0) > p^*$



Summary

Why do we lie?

- If you convince $> z'$ people that you are King,
- then they will help you to subjugate z'' people.



Outline

El Farol Bar, Santa Fe NM

El Farol Problem: Minority Game

- capacity: 60 places
- attraction: music nights
- customers: 100 music fans
  - # visitors $\leq 60 \implies$ pleasant
  - # visitors $> 60 \implies$ unpleasant
- goal of the game: visit El Farol when # visitors ≤ 60

Minority Game

- players: $i = 1, 2, \ldots, 100$
- moves: $A_i = \{Y, N\}$, for all i
- payoffs:

$$u_i(a) \quad = \quad \begin{cases} 1 & \text{if } \#\{k | a_k = a_i\} \leq 60 \\ -1 & \text{if } \#\{k | a_k = a_i\} > 60 \end{cases}$$

Minority Game

Exercise

Analyze Nash equilibria in this game.

Minority Game

Negative feedback

- The members of the majority have a *joint* incentive to switch.
  - "No one goes to El Farol. It's too busy."
- The Nash equilibria are *unstable*.

Recall: Network effects

- Let p^* be the fixed (average) production cost.
- Suppose that x **believes** that z-part of the market has adopted Γ (which may not be true).
- The true market adoption (depending on the belief z) is

$$g(z) \quad = \quad q\left(\frac{p^*}{f(z)}\right)$$

because $r^{-1} = q$.

Negative network effects

Given

- fixed production price p^*
- reserved price $r(z) = 1 - z$, demand $q(z) = r^{-1}(z) = 1 - z$
- network effect $f(z) = \begin{cases} z & \text{if } z \leq .6 \\ 1 - z & \text{if } z > .6 \end{cases}$

the true adoption is $\widehat{z} = g(z) = \begin{cases} 0 & \text{if } z \leq p^* \\ 1 - \frac{p^*}{z} & p^* < z \leq .6 \\ 1 - \frac{p^*}{1-z} & .6 < z \end{cases}$

Dynamics of El Farol Bar

ongoing research